MODELING CURRENT DISTRIBUTION IN A MAGNETOHYDRODYNAMIC CHANNEL WITH ALLOWANCE FOR CONDUCTIVITY ANISOTROPY

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1. A number of authors [1-3] have computed the electric potential distribution and the current density distribution in a moving fluid with isotropic conductivity. In this connection, it is of interest to consider the possibility of electrically modeling such problems on materials with a suitable conductivity tensor. Such a conductor, for example, is polycrystalline bismuth placed in a magnetic field.

Below it is assumed that the magnetic field and the velocity field are given. The flow of fluid with tensor conductivity is surrounded by walls consisting of insulators and perfect conductors. The latter may be electrodes across which certain electric potentials are applied. All the formulas are written in the MKSA system of units.

Let us consider the conditions that must be fulfilled if for a stationary model without distributed current sources the current density distribution is to be the same as in a system with a moving fluid.

The equation of current continuity and Ohm's law can be written in the following form:

$$\operatorname{div} \mathbf{j} = 0, \qquad \mathbf{j} = -\sigma \operatorname{grad} V. \tag{1.1}$$

Here $\mathbf{j}$ is the current density, $V$ is the electric potential, and $\sigma$ is the electron conductivity tensor which in the coordinate system with the magnetic field directed along the z axis has the following form:

$$\sigma = \frac{\sigma_0}{1+\mu^2B^2}\begin{pmatrix} 1 & -\mu B & 0 \\ \mu B & 1 & 0 \\ 0 & 0 & 1+\mu^2B^2 \end{pmatrix}. \tag{1.2}$$

Here $\mu$ is the Hall mobility, $\sigma_0$ is the conductivity along the magnetic field, and $\mu B = \omega\tau$. The analogous equations for a system with a moving fluid have the form

$$\operatorname{div} \mathbf{j} = 0, \quad \mathbf{j} = \sigma(-\operatorname{grad} V + \mathbf{v}\times\mathbf{B}). \tag{1.3}$$

Here $\mathbf{v}$ is the fluid velocity, and $\mathbf{B}$ is the magnetic induction vector.

The second equation of (1.3) can be reduced to the form of the second equation of (1.1), if the vector

$$\mathbf{E}^* = -\operatorname{grad} V + \mathbf{v}\times\mathbf{B}$$

can be represented as the gradient of some quantity W, which must also be made to correspond with the electric potential in the mode. This in turn, leads to the condition

$$\operatorname{rot}\mathbf{E}^* = (\mathbf{B}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B} - \mathbf{B}\operatorname{div}\mathbf{v} = 0. \tag{1.4}$$

In writing Eq. (1.4) use was made of the equation $\operatorname{div}\mathbf{B} = 0$.

Condition (1.4) imposes a constraint on the configuration of the magnetic fields and velocity fields for which modeling using models without distributed current sources is possible. The simplest case of fulfillment of this condition is that in which each term of (1.4) is equal to zero. If the first term is equal to zero, we get the condition of constancy of the velocity vector at the lines of force. If the second term is equal to zero, the magnetic induction vector must be constant along the streamlines. If the last term is equal to zero, then the fluid must be incompressible.

If condition (1.4) is satisfied, then at any point M we can determine the potential W correct to a constant

$$W(M) = V(M) - \int_{M_0}^{M} (\mathbf{v}\times\mathbf{B})\cdot d\mathbf{l}. \tag{1.5}$$

Here $d\mathbf{l}$ is an element of the integration path.

Ohm's law can now be written in the same form as for the model

$$\mathbf{j} = -\sigma \operatorname{grad} V. \tag{1.6}$$

At the boundary with an insulator the boundary conditions for the potential W and the electric potential in the case of the model are also the same, which follows from the fact that the normal component of the current density is zero and from the identical form of Ohm's law (1.6). Cases are possible where on a surface that is a good conductor the auxiliary potential W is constant together with V. Then, if at the electrodes we specify electric potentials proportional to the corresponding values of the potential W for the system with a moving fluid, the distributions of both potentials will be similar. The current density distributions will also be similar. In this case the quantity $\mu B = \omega\tau$ must be the same in each case.

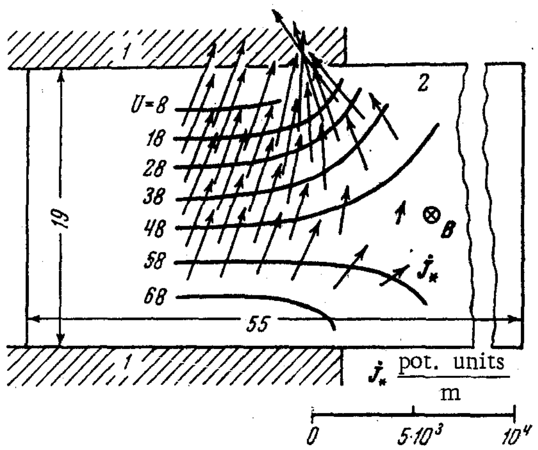

Fig. 1

2. The fringe effects were modeled in a rectangular channel with crossed fields and continuous electrodes. The magnetic field was assumed to be uniform. Owing to the nonuniformity of the velocity field over the section of the channel

$$(\mathbf{B}\cdot\nabla)\mathbf{v} \neq 0$$

(the velocity varies along the lines of force). However, as shown in [4], in the given case the three-dimensional problem can be reduced to a plane problem for quantities averaged along the magnetic lines of force. This plane problem is solved by modeling (provided that the other two terms in (1.4) are equal to zero). An analogous plane problem was modeled in [5] for the case of scalar conductivity.

In accordance with (1.5), the potential W for the plane case is written in the form

$$W = V + B\Phi \qquad (\partial\Phi / \partial y = v_x, \quad -\partial\Phi / \partial x = v_y),$$

To the two electrodes there corresponds a potential difference W equal to

$$W_0 = V_0 + B\Phi_0 \,. \tag{2.2}$$

Here $V_0$ is the electric potential difference between the electrodes, $\Phi_0$ is the rate of fluid flow referred to unit height of the channel along the magnetic field.

In the problem with two electrodes all possible distributions of the potential W and current density are independent of the velocity profile in a direction perpendicular to the magnetic field.

The arrangement of the model is clear from Fig. 1. The fluid was modeled with a plate of polycrystalline bismuth with Hall mobility $\mu = 4000\ \mathrm{cm^2/V\text{-}sec}$ at room temperature. In order to increase the mobility, we cooled the model to $-20°C$. In a magnetic field of the order of 5000 gauss we obtained a value of the product $\mu B = 0.3\text{-}0.4$. This quantity was measured under the same conditions as for the model on a special specimen of the same bismuth. The electrodes were made of copper with a cross section large enough for simulation of the equipotential surface.

The device worked on alternating current of commerical frequency. This made it particularly simple to automate the process of obtaining a picture of the field and supplying the model. We were able to use alternating current because of the linear character of equations (1.1) and the boundary conditions. At any point of the model the electric potential will vary in accordance with the same law as at the electrodes. Accordingly, the equipotential lines will retain their position at any moment of time.

The potential distribution on the surface of the bismuth plate was recorded by a compensation method using a bridge circuit exactly like that used in working with electrolytic baths. The registration of equipotentials was semiautomatic [6]. The voltage was picked up from the surface of the model by a probe. By means of a special circuit a capacitor was discharged through the probe when it crossed a certain equipotential, and a mark was made on the surface of the model. In this way we eliminated errors connected with transfer of the motion of the probe and the configuration of the model to the drawing. An electronic circuit registered passage of the phase of the probe signal through the value $\pi/2$ or of the amplitude through zero when an equipotential was crossed.

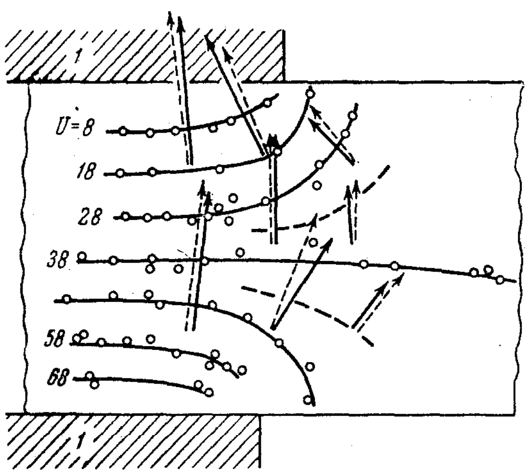

Fig. 2

Figure 1 shows an example of a potential field recorded in this way in relative units together with the calculated current density vector field for $\mu B = 0.4$. Here 1 denotes the copper electrode, 2 the bismuth plate. The current density was calculated for $\sigma_0 = 1$ from the formula

$$\mathbf{j}_* = \frac{1}{1+\mu^2 B^2}\begin{pmatrix} 1 & -\mu B \\ \mu B & 1 \end{pmatrix} \operatorname{grad} V. \tag{2.3}$$

The quantity j has the same dimension as the field strength. Conversion to the current density in the device with a moving fluid can be based on the formula

$$\mathbf{j} = \mathbf{j}_* \sigma_0 W_0 / U_{\text{OM}} M, \tag{2.4}$$

where $\sigma_0$ is the conductivity of the fluid and M is the ratio of the dimensions of the device to the dimensions of the model.

In estimating the accuracy of the method it should be noted that the similarity of the current density distributions is exact if the above-mentioned conditions are fulfilled. In order to estimate the accuracy of construction of the field pattern, the accuracy of the model, and the effect of the finite length of the model "channel," we recorded the field for $B = 0$; the remaining conditions were kept the same. The field pattern and the results of the comparison are presented in Fig. 2. The maximum scatter of the equipotential marks did not exceed 3% of the electrode voltage. The region of operation of the automatic marker did not exceed 0.1-0.2%. The scatter of the experimental points was conditioned by the imperfect model-probe contact and the variation of the model temperature. The field strength found from the potential distribution (solid arrows) was compared with that computed from the formulas of [7] (broken arrows). The error in determining the field strength, referred to its value in the uniform field region, was, on the average, 10% for the points indicated on the drawing. This value should be taken as a measure of the accuracy of the method, although in a number of cases the accuracy may be overestimated.

The use of modeling makes it possible to reduce the labor of problem solving. Moreover, problems with tensor conductivity depending on the coordinates can be solved very simply.

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